

EXEMPLAR SOLUTIONS MATHS

Chapter 1 : Relations and Functions

:

Class
12



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Chapter 1: Relations and Functions

Exercise 1.3

Short Answer (S.A.)

1. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows:

$R = \{(a, a), (b, c), (a, b)\}$.

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Solution:

Given relation, $R = \{(a, a), (b, c), (a, b)\}$

To make R as reflexive we should add (b, b) and (c, c) to R . Also, to make R as transitive we should add (a, c) to R .

Hence, the minimum number of ordered pairs to be added are (b, b) , (c, c) and (a, c) i.e. 3.

2. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then, write D .

Solution:

Given, $f(x) = \sqrt{25 - x^2}$

The function is defined if $25 - x^2 \geq 0$

So, $x^2 \leq 25$

$-5 \leq x \leq 5$

Therefore, the domain of the given function is $[-5, 5]$

3. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2, \forall x \in \mathbb{R}$, respectively. Then, find $g \circ f$.

Solution:

Given,

$f(x) = 2x + 1$ and $g(x) = x^2 - 2, \forall x \in \mathbb{R}$

Thus, $g \circ f = g(f(x))$

$$= g(2x + 1)$$

$$= (2x + 1)^2 - 2$$

$$= 4x^2 + 4x + 1 - 2$$

$$= 4x^2 + 4x - 1$$

4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 2x - 3, \forall x \in \mathbb{R}$. write f^{-1} .

Solution:

Given function,

$f(x) = 2x - 3, \forall x \in \mathbb{R}$

Let $y = 2x - 3$

$x = (y + 3)/2$

Thus,

$f^{-1}(x) = (x + 3)/2$

5. If $A = \{a, b, c, d\}$ and the function $f = \{(a, b), (b, d), (c, a), (d, c)\}$, write f^{-1} .

Solution:

Given,

$$A = \{a, b, c, d\} \text{ and } f = \{(a, b), (b, d), (c, a), (d, c)\}$$

So,

$$f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$$

6. If $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined by $f(x) = x^2 - 3x + 2$, write $f(f(x))$.

Solution:

Given, $f(x) = x^2 - 3x + 2$

Then,

$$\begin{aligned}
 f(f(x)) &= f(x^2 - 3x + 2) \\
 &= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2, \\
 &= x^4 + 9x^2 + 4 - 6x^3 + 4x^2 - 12x - 3x^2 + 9x - 6 + 2 \\
 &= x^4 - 6x^3 + 10x^2 - 3x
 \end{aligned}$$

Thus,

$$f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$$

7. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β .

Solution:

Given, $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$

It's seen that every element of domain has a unique image. So, g is function.

Now, also given that $g(x) = \alpha x + \beta$

So, we have

$$g(1) = \alpha(1) + \beta = 1$$

$$\alpha + \beta = 1 \dots\dots\dots (i)$$

$$\text{And, } g(2) = \alpha(2) + \beta = 3$$

$$2\alpha + \beta = 3 \dots\dots\dots (ii)$$

Solving (i) and (ii), we have

$$\alpha = 2 \text{ and } \beta = -1$$

$$\text{Therefore, } g(x) = 2x - 1$$

8. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

(i) $\{(x, y): x \text{ is a person, } y \text{ is the mother of } x\}$.

(ii) $\{(a, b): a \text{ is a person, } b \text{ is an ancestor of } a\}$.

Solution:

(i) Given, $\{(x, y): x \text{ is a person, } y \text{ is the mother of } x\}$

It's clearly seen that each person 'x' has only one biological mother.

Hence, the above set of ordered pairs make a function.

Now more than one person may have same mother. Thus, the function is many-many one and surjective.

(ii) Given, $\{(a, b): a \text{ is a person, } b \text{ is an ancestor of } a\}$
 It's clearly seen that any person 'a' has more than one ancestors.
 Thus, it does not represent a function.

9. If the mappings f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, write $f \circ g$.

Solution:

Given,
 $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$
 Now,
 $f \circ g(2) = f(g(2)) = f(3) = 5$
 $f \circ g(5) = f(g(5)) = f(1) = 2$
 $f \circ g(1) = f(g(1)) = f(3) = 5$
 Thus,
 $f \circ g = \{(2, 5), (5, 2), (1, 5)\}$

10. Let C be the set of complex numbers. Prove that the mapping $f: C \rightarrow R$ given by $f(z) = |z|, \forall z \in C$, is neither one-one nor onto.

Solution:

Given, $f: C \rightarrow R$ such that $f(z) = |z|, \forall z \in C$
 Now, let take $z = 6 + 8i$
 Then,
 $f(6 + 8i) = |6 + 8i| = \sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$
 And, for $z = 6 - 8i$
 $f(6 - 8i) = |6 - 8i| = \sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$
 Hence, $f(z)$ is many-one.
 Also, $|z| \geq 0, \forall z \in C$
 But the co-domain given is ' R '
 Therefore, $f(z)$ is not onto.

11. Let the function $f: R \rightarrow R$ be defined by $f(x) = \cos x, \forall x \in R$. Show that f is neither one-one nor onto.

Solution:

We have,
 $f: R \rightarrow R, f(x) = \cos x$
 Now,
 $f(x_1) = f(x_2)$
 $\cos x_1 = \cos x_2$
 $x_1 = 2n\pi \pm x_2, n \in Z$
 It's seen that the above equation has infinite solutions for x_1 and x_2
 Hence, $f(x)$ is many one function.
 Also the range of $\cos x$ is $[-1, 1]$, which is subset of given co-domain R .

Therefore, the given function is not onto.

12. Let $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$. Find whether the following subsets of $X \times Y$ are functions from X to Y or not.

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$ (ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $h = \{(1, 4), (2, 5), (3, 5)\}$ (iv) $k = \{(1, 4), (2, 5)\}$.

Solution:

Given, $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$

So, $X \times Y = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

f is not a function as $f(1) = 4$ and $f(1) = 5$

Hence, pre-image '1' has not unique image.

(ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

It's seen clearly that g is a function in which each element of the domain has unique image.

(iii) $h = \{(1, 4), (2, 5), (3, 5)\}$

It's seen clearly that h is a function as each pre-image with a unique image.

And, function h is many-one as $h(2) = h(3) = 5$

(iv) $k = \{(1, 4), (2, 5)\}$

Function k is not a function as '3' has not any image under the mapping.

13. If functions $f: A \rightarrow B$ and $g: B \rightarrow A$ satisfy $g \circ f = I_A$, then show that f is one-one and g is onto.

Solution:

Given,

$f: A \rightarrow B$ and $g: B \rightarrow A$ satisfy $g \circ f = I_A$

It's clearly seen that function ' g ' is inverse of ' f '.

So, ' f ' has to be one-one and onto.

Hence, ' g ' is also one-one and onto.

14. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 1/(2 - \cos x) \forall x \in \mathbb{R}$. Then, find the range of f .

Solution:

Given,

$f(x) = 1/(2 - \cos x) \forall x \in \mathbb{R}$

Let $y = 1/(2 - \cos x)$

$2y - y \cos x = 1$

$\cos x = (2y - 1)/y$

$\cos x = 2 - 1/y$

Now, we know that $-1 \leq \cos x \leq 1$

So,

$-1 \leq 2 - 1/y \leq 1$

$-3 \leq -1/y \leq -1$

$$1 \leq -1/y \leq 3$$

$$1/3 \leq y \leq 1$$

Thus, the range of the given function is $[1/3, 1]$.

15. Let n be a fixed positive integer. Define a relation R in Z as follows: $\forall a, b \in Z$, aRb if and only if $a - b$ is divisible by n . Show that R is an equivalence relation.

Solution:

Given $\forall a, b \in Z$, aRb if and only if $a - b$ is divisible by n .

Now, for

$aRa \Rightarrow (a - a)$ is divisible by n , which is true for any integer a as '0' is divisible by n .

Thus, R is reflexive.

Now, aRb

So, $(a - b)$ is divisible by n .

$\Rightarrow -(b - a)$ is divisible by n .

$\Rightarrow (b - a)$ is divisible by n

$\Rightarrow bRa$

Thus, R is symmetric.

Let aRb and bRc

Then, $(a - b)$ is divisible by n and $(b - c)$ is divisible by n .

So, $(a - b) + (b - c)$ is divisible by n .

$\Rightarrow (a - c)$ is divisible by n .

$\Rightarrow aRc$

Thus, R is transitive.

So, R is an equivalence relation.

Long Answer (L.A.)

16. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being:

(a) reflexive, transitive but not symmetric

(b) symmetric but neither reflexive nor transitive

(c) reflexive, symmetric and transitive.

Solution:

Given that, $A = \{1, 2, 3\}$.

(i) Let $R_1 = \{(1, 1), (1, 2), (1, 3), (2, 3), (2, 2), (1, 3), (3, 3)\}$

R_1 is reflexive as $(1, 1)$, $(2, 2)$ and $(3, 3)$ lie in R_1 .

R_1 is transitive as $(1, 2) \in R_1$, $(2, 3) \in R_1 \Rightarrow (1, 3) \in R_1$

Now, $(1, 2) \in R_1 \Rightarrow (2, 1) \notin R_1$.

(ii) Let $R_2 = \{(1, 2), (2, 1)\}$

Now, $(1, 2) \in R_2$, $(2, 1) \in R_2$

So, it is symmetric,

And, clearly R_2 is not reflexive as $(1, 1) \notin R_2$

Also, R_2 is not transitive as $(1, 2) \in R_2$, $(2, 1) \in R_2$ but $(1, 1) \notin R_2$

(iii) Let $R_3 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

R_3 is reflexive as $(1, 1)$, $(2, 2)$ and $(3, 3) \in R_1$

R_3 is symmetric as $(1, 2), (1, 3), (2, 3) \in R_1 \Rightarrow (2, 1), (3, 1), (3, 2) \in R_1$

Therefore, R_3 is reflexive, symmetric and transitive.

17. Let R be relation defined on the set of natural number N as follows:

$R = \{(x, y): x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R . Also verify whether R is reflexive, symmetric and transitive.

Solution:

Given function: $R = \{(x, y): x \in N, y \in N, 2x + y = 41\}$.

So, the domain = $\{1, 2, 3, \dots, 20\}$ [Since, $y \in N$]

Finding the range, we have

$R = \{(1, 39), (2, 37), (3, 35), \dots, (19, 3), (20, 1)\}$

Thus, Range of the function = $\{1, 3, 5, \dots, 39\}$

R is not reflexive as $(2, 2) \notin R$ as $2 \times 2 + 2 \neq 41$

Also, R is not symmetric as $(1, 39) \in R$ but $(39, 1) \notin R$

Further R is not transitive as $(11, 19) \notin R, (19, 3) \notin R$; but $(11, 3) \notin R$.

Thus, R is neither reflexive nor symmetric and nor transitive.

18. Given $A = \{2, 3, 4\}$, $B = \{2, 5, 6, 7\}$. Construct an example of each of the following:

(a) an injective mapping from A to B

(b) a mapping from A to B which is not injective

(c) a mapping from B to A .

Solution:

Given, $A = \{2, 3, 4\}$, $B = \{2, 5, 6, 7\}$

(i) Let $f: A \rightarrow B$ denote a mapping

$f = \{(x, y): y = x + 3\}$ or

$f = \{(2, 5), (3, 6), (4, 7)\}$, which is an injective mapping.

(ii) Let $g: A \rightarrow B$ denote a mapping such that $g = \{(2, 2), (3, 2), (4, 5)\}$, which is not an injective mapping.

(iii) Let $h: B \rightarrow A$ denote a mapping such that $h = \{(2, 2), (5, 3), (6, 4), (7, 4)\}$, which is one of the mapping from B to A .

19. Give an example of a map

(i) which is one-one but not onto

(ii) which is not one-one but onto

(iii) which is neither one-one nor onto.

Solution:

(i) Let $f: N \rightarrow N$, be a mapping defined by $f(x) = x^2$

For $f(x_1) = f(x_2)$

Then, $x_1^2 = x_2^2$

$x_1 = x_2$ (Since $x_1 + x_2 = 0$ is not possible)

Further ' f ' is not onto, as for $1 \in N$, there does not exist any x in N such that $f(x) = 2x + 1$.

(ii) Let $f: \mathbb{R} \rightarrow [0, \infty)$, be a mapping defined by $f(x) = |x|$
 Then, it's clearly seen that $f(x)$ is not one-one as $f(2) = f(-2)$.
 But $|x| \geq 0$, so range is $[0, \infty]$.
 Therefore, $f(x)$ is onto.

(iii) Let $f: \mathbb{R} \rightarrow \mathbb{R}$, be a mapping defined by $f(x) = x^2$
 Then clearly $f(x)$ is not one-one as $f(1) = f(-1)$. Also range of $f(x)$ is $[0, \infty)$.
 Therefore, $f(x)$ is neither one-one nor onto.

20. Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. Let $f: A \rightarrow B$ be defined by $f(x) = x - 2/x - 3 \forall x \in A$. Then show that f is bijective.

Solution:

Given,

$$A = \mathbb{R} - \{3\}, B = \mathbb{R} - \{1\}$$

And,

$$f: A \rightarrow B \text{ be defined by } f(x) = x - 2/x - 3 \forall x \in A$$

$$\text{Hence, } f(x) = (x - 3 + 1)/(x - 3) = 1 + 1/(x - 3)$$

$$\text{Let } f(x_1) = f(x_2)$$

$$1 + \frac{1}{x_1 - 3} = 1 + \frac{1}{x_2 - 3}$$

$$\frac{1}{x_1 - 3} = \frac{1}{x_2 - 3}$$

$$x_1 = x_2$$

So, $f(x)$ is an injective function.

$$\text{Now let } y = (x - 2)/(x - 3)$$

$$x - 2 = xy - 3y$$

$$x(1 - y) = 2 - 3y$$

$$x = (3y - 2)/(y - 1)$$

$$y \in \mathbb{R} - \{1\} = B$$

Thus, $f(x)$ is onto or surjective.

Therefore, $f(x)$ is a bijective function.

21. Let $A = [-1, 1]$. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

(i) $f(x) = x/2$

(ii) $g(x) = |x|$

(iii) $h(x) = x|x|$

(iv) $k(x) = x^2$

Solution:

$$\text{Given, } A = [-1, 1]$$

(i) $f: [-1, 1] \rightarrow [-1, 1], f(x) = x/2$

$$\text{Let } f(x_1) = f(x_2)$$

$$x_1/2 = x_2$$

So, $f(x)$ is one-one.

Also $x \in [-1, 1]$

$$x/2 = f(x) = [-1/2, 1/2]$$

Hence, the range is a subset of co-domain 'A'

So, $f(x)$ is not onto.

Therefore, $f(x)$ is not bijective.

$$(ii) g(x) = |x|$$

$$\text{Let } g(x_1) = g(x_2)$$

$$|x_1| = |x_2|$$

$$x_1 = \pm x_2$$

So, $g(x)$ is not one-one

Also $g(x) = |x| \geq 0$, for all real x

Hence, the range is $[0, 1]$, which is subset of co-domain 'A'

So, $f(x)$ is not onto.

Therefore, $f(x)$ is not bijective.

$$(iii) h(x) = x|x|$$

$$\text{Let } h(x_1) = h(x_2)$$

$$x_1|x_1| = x_2|x_2|$$

If $x_1, x_2 > 0$

$$x_1^2 = x_2^2$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 = x_2 \text{ (as } x_1 + x_2 \neq 0)$$

Similarly for $x_1, x_2 < 0$, we have $x_1 = x_2$

It's clearly seen that for x_1 and x_2 of opposite sign, $x_1 \neq x_2$.

Hence, $f(x)$ is one-one.

For $x \in [0, 1]$, $f(x) = x^2 \in [0, 1]$

For $x < 0$, $f(x) = -x^2 \in [-1, 0)$

Hence, the range is $[-1, 1]$.

So, $h(x)$ is onto.

Therefore, $h(x)$ is bijective.

$$(iv) k(x) = x^2$$

$$\text{Let } k(x_1) = k(x_2)$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

Therefore, $k(x)$ is not one-one.

22. Each of the following defines a relation on \mathbb{N} :

(i) x is greater than y , $x, y \in \mathbb{N}$

(ii) $x + y = 10$, $x, y \in \mathbb{N}$

(iii) x is square of an integer $x, y \in \mathbb{N}$

(iv) $x + 4y = 10$, $x, y \in \mathbb{N}$.

Determine which of the above relations are reflexive, symmetric and transitive.

Solution:

(i) Given, x is greater than y ; $x, y \in \mathbb{N}$

If $(x, x) \in R$, then $x > x$, which is not true for any $x \in \mathbb{N}$.

Thus, R is not reflexive.

Let $(x, y) \in R$

$\Rightarrow xRy$

$\Rightarrow x > y$

So, $y > x$ is not true for any $x, y \in \mathbb{N}$

Hence, R is not symmetric.

Let xRy and yRz

$\Rightarrow x > y$ and $y > z$

$\Rightarrow x > z$

$\Rightarrow xRz$

Hence, R is transitive.

(ii) $x + y = 10$; $x, y \in \mathbb{N}$

Thus,

$R = \{(x, y); x + y = 10, x, y \in \mathbb{N}\}$

$R = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$

It's clear $(1, 1) \notin R$

So, R is not reflexive.

$(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric.

Now $(1, 9) \in R, (9, 1) \in R$, but $(1, 1) \notin R$

Therefore, R is not transitive.

(iii) Given, xy is square of an integer $x, y \in \mathbb{N}$

$R = \{(x, y) : xy \text{ is a square of an integer } x, y \in \mathbb{N}\}$

It's clearly $(x, x) \in R, \forall x \in \mathbb{N}$

As x^2 is square of an integer for any $x \in \mathbb{N}$

Thus, R is reflexive.

If $(x, y) \in R \Rightarrow (y, x) \in R$

So, R is symmetric.

Now, if xy is square of an integer and yz is square of an integer.

Then, let $xy = m^2$ and $yz = n^2$ for some $m, n \in \mathbb{Z}$

$x = m^2/y$ and $z = x^2/y$

$xz = m^2n^2/y^2$, which is square of an integer.

Thus, R is transitive.

(iv) $x + 4y = 10$; $x, y \in \mathbb{N}$

$R = \{(x, y); x + 4y = 10; x, y \in \mathbb{N}\}$

$R = \{(2, 2), (6, 1)\}$

It's clearly seen $(1, 1) \notin R$

Hence, R is not symmetric.

$$(x, y) \in R \Rightarrow x + 4y = 10$$

$$\text{And } (y, z) \in R \Rightarrow y + 4z = 10$$

$$\Rightarrow x - 16z = -30$$

$$\Rightarrow (x, z) \notin R$$

Therefore, R is not transitive.

23. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalent class $[(2, 5)]$.

Solution:

Given, $A = \{1, 2, 3, \dots, 9\}$ and $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d) \in A \times A$.

Let $(a, b) R (a, b)$

So, $a + b = b + a$, $\forall a, b \in A$ which is true for any $a, b \in A$.

Thus, R is reflexive.

Let $(a, b) R (c, d)$

Then,

$$a + d = b + c$$

$$c + b = d + a$$

$$(c, d) R (a, b)$$

Thus, R is symmetric.

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$a + d = b + c \text{ and } c + f = d + e$$

$$a + d = b + c \text{ and } d + e = c + f$$

$$(a + d) - (d + e) = (b + c) - (c + f)$$

$$a - e = b - f$$

$$a + f = b + e$$

$$(a, b) R (e, f)$$

So, R is transitive.

The equivalence class $[(2, 5)] = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

Therefore, R is an equivalence relation.

24. Using the definition, prove that the function $f : A \rightarrow B$ is invertible if and only if f is both one-one and onto.

Solution:

Let $f : A \rightarrow B$ be many-one function.

Let $f(a) = p$ and $f(b) = p$

So, for inverse function we will have $f^{-1}(p) = a$ and $f^{-1}(p) = b$

Thus, in this case inverse function is not defined as we have two images 'a and b' for one pre-image 'p'. But for f to be invertible it must be one-one.

Now, let $f : A \rightarrow B$ is not onto function.

Let $B = \{p, q, r\}$ and range of f be $\{p, q\}$.

Here image 'r' has not any pre-image, which will have no image in set A .

And for f to be invertible it must be onto.

Thus, ' f ' is invertible if and only if ' f ' is both one-one and onto.

A function $f = X \rightarrow Y$ is invertible iff f is a bijective function.

25. Functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find

(i) $f \circ g$ (ii) $g \circ f$ (iii) $f \circ f$ (iv) $g \circ g$

Solution:

Given, $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$

(i) $f \circ g = f(g(x))$

$$= f(2x - 3)$$

$$= (2x - 3)^2 + 3(2x - 3) + 1$$

$$= 4x^2 + 9 - 12x + 6x - 9 + 1$$

$$= 4x^2 - 6x + 1$$

(ii) $g \circ f = g(f(x))$

$$= g(x^2 + 3x + 1)$$

$$= 2(x^2 + 3x + 1) - 3$$

$$= 2x^2 + 6x - 1$$

(iii) $f \circ f = f(f(x))$

$$= f(x^2 + 3x + 1)$$

$$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$$

$$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$$

$$= x^4 + 6x^3 + 14x^2 + 15x + 5$$

(iv) $g \circ g = g(g(x))$

$$= g(2x - 3)$$

$$= 2(2x - 3) - 3$$

$$= 4x - 6 - 3$$

$$= 4x - 9$$

26. Let $*$ be the binary operation defined on \mathbb{Q} . Find which of the following binary operations are commutative

(i) $a * b = a - b \forall a, b \in \mathbb{Q}$

(ii) $a * b = a^2 + b^2 \forall a, b \in \mathbb{Q}$

(iii) $a * b = a + ab \forall a, b \in \mathbb{Q}$

(iv) $a * b = (a - b)^2 \forall a, b \in \mathbb{Q}$

Solution:

Given that $*$ is a binary operation defined on \mathbb{Q} .

(i) $a * b = a - b, \forall a, b \in \mathbb{Q}$ and $b * a = b - a$

So, $a * b \neq b * a$

Thus, $*$ is not commutative.

(ii) $a * b = a^2 + b^2$

$$b * a = b^2 + a^2$$

Thus, $*$ is commutative.

(iii) $a * b = a + ab$

$$b * a = b + ab$$

So clearly, $a + ab \neq b + ab$

Thus, $*$ is not commutative.

(iv) $a * b = (a - b)^2, \forall a, b \in \mathbb{Q}$

$$b * a = (b - a)^2$$

Since, $(a - b)^2 = (b - a)^2$
Thus, $*$ is commutative.

27. Let $*$ be binary operation defined on R by $a * b = 1 + ab, \forall a, b \in R$. Then the operation $*$ is

- (i) commutative but not associative**
- (ii) associative but not commutative**
- (iii) neither commutative nor associative**
- (iv) both commutative and associative**

Solution:

(i) Given that $*$ is a binary operation defined on R by $a * b = 1 + ab, \forall a, b \in R$

So, we have $a * b = ab + 1 = b * a$

So, $*$ is a commutative binary operation.

Now, $a * (b * c) = a * (1 + bc) = 1 + a(1 + bc) = 1 + a + abc$

Also,

$(a * b) * c = (1 + ab) * c = 1 + (1 + ab)c = 1 + c + abc$

Thus, $a * (b * c) \neq (a * b) * c$

Hence, $*$ is not associative.

Therefore, $*$ is commutative but not associative.

Objective Type Questions

Choose the correct answer out of the given four options in each of the Exercises from 28 to 47 (M.C.Q.)

28. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \forall a, b \in T$. Then R is

- (A) reflexive but not transitive**
- (B) transitive but not symmetric**
- (C) equivalence**
- (D) none of these**

Solution:

(C) equivalence

Given aRb , if a is congruent to $b, \forall a, b \in T$.

Then, we have $aRa \Rightarrow a$ is congruent to a ; which is always true.

So, R is reflexive.

Let $aRb \Rightarrow a \sim b$

$b \sim a$

bRa

So, R is symmetric.

Let aRb and bRc

$a \sim b$ and $b \sim c$

$a \sim c$

aRc

So, R is transitive.

Therefore, R is equivalence relation.

29. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b . Then R is

- (A) symmetric but not transitive
(C) neither symmetric nor transitive

- (B) transitive but not symmetric
(D) both symmetric and transitive

Solution:

(B) transitive but not symmetric

$aRb \Rightarrow a$ is brother of b .

This does not mean b is also a brother of a as b can be a sister of a .

Thus, R is not symmetric.

$aRb \Rightarrow a$ is brother of b .

and $bRc \Rightarrow b$ is brother of c .

So, a is brother of c .

Therefore, R is transitive.

30. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

- (A) 1 (B) 2
(C) 3 (D) 5

Solution:

(D) 5

Given, set $A = \{1, 2, 3\}$

Now, the number of equivalence relations as follows

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$

$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$

Thus, maximum number of equivalence relation is '5'.

31. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

- (A) reflexive (B) transitive
(C) symmetric (D) none of these

Solution:

(D) none of these

R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$

Hence, its clear that R is not reflexive, transitive and symmetric.

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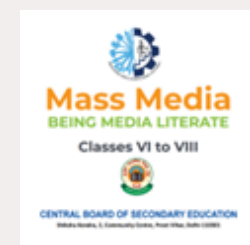
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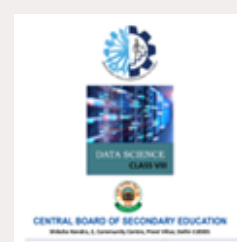
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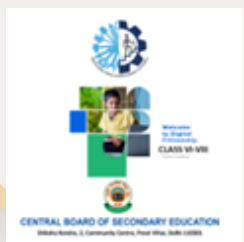
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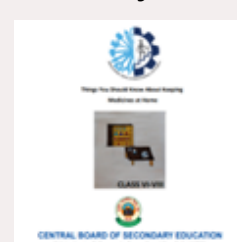
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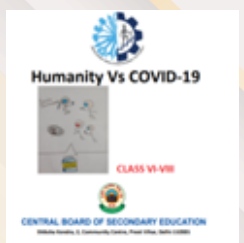
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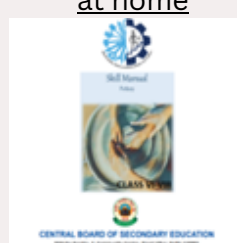
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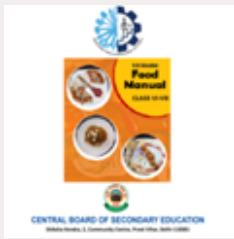
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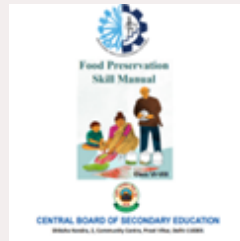
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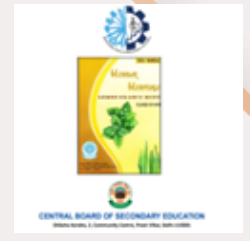
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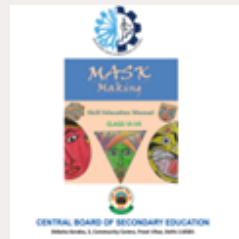
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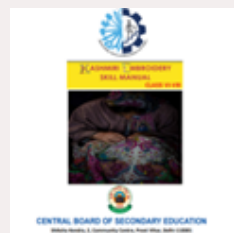
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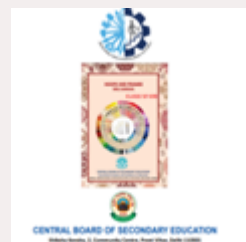
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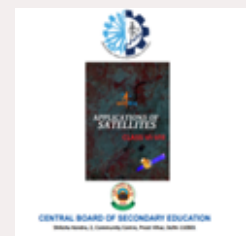
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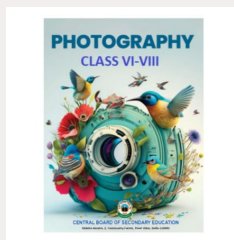
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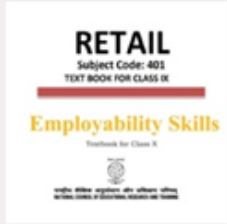


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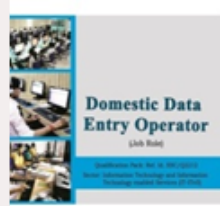


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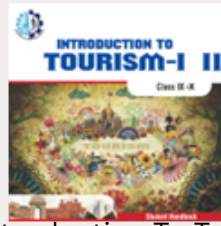
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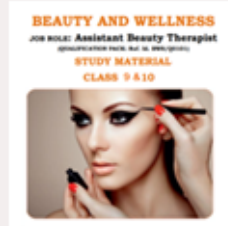
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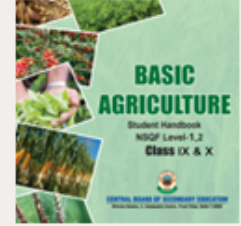
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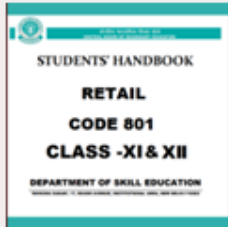


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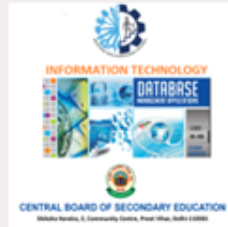


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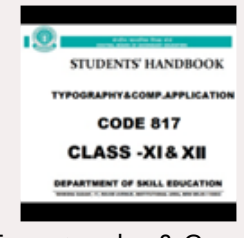
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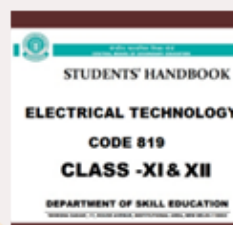
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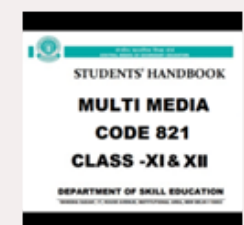
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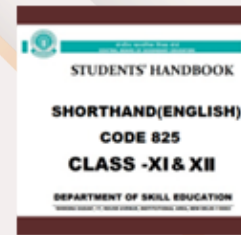
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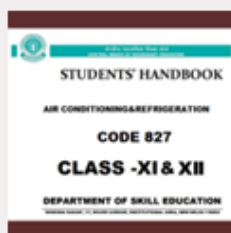
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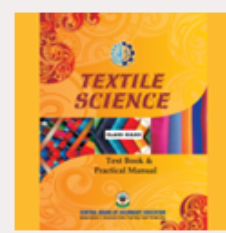
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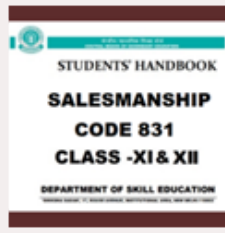
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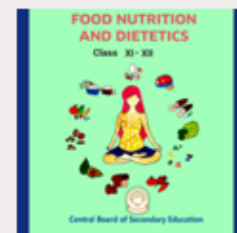
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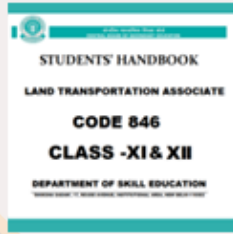
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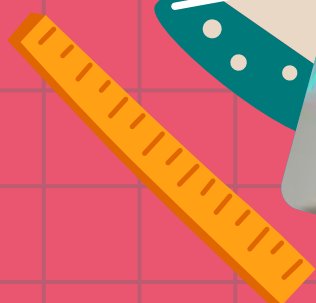
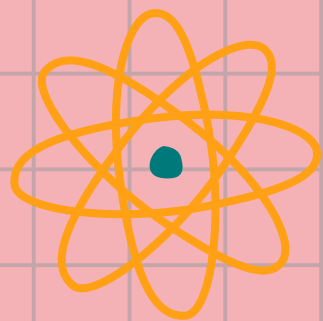
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